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DETERMINATION OF THE STRESS FIELD OF THE BRITTLE ROCK CAUSED BY QUASI-STATIC LOAD

Abstract

In the paper the mathematical model has been proposed according of which the rock is considered as an orthotropic half-plane. On the boundary of this half-plane the loads, moving at a constant velocity, exert a pressure. The problem was solved by means of the theory of analytical functions.

Keywords

The rock, tensor, mechanical fracture.

1 INTRODUCTION

The rock failure is a highly complex problem, accompanied by its sides, which among the mechanical destruction is one of the most important problem. Solution of fracture mechanics problems often enables to explain many phenomena and predict failure of bodies of various shapes.

2 THE MAIN PART

The type of distribution of the load, acting on the surface, is of great importance in the course of the research of the processes of mechanical failure of the rocks. We will study the elastic-stressed state of petrean rocks whereas the load, moving by constant velocity C and disturbed by definite regularity $q_i(x),(i=1,2...,n)$, acts on the boundary.

Taking into the consideration the fact that the failure of petrean rock proceeds at relatively small deformations, then we can use the methods of classical theory of elasticity. Also the fact should be taken into account that the effect of acting load on the rock surface is gradually reduced with increase of the rock thickness (z) and in the long run completely disappears. For reasonably large z the weight of upper layers of the rock effects on the stress field. So that the attack of the load, acting on the rock surface, is considerable for the layer of small thickness (in comparison with other dimensions). Hence it may be thought to a definite approximation that the rock presents the orthotropic half-plane. The directions of its principal axes are coincident with the directions of the coordinate axes. From all above-mentioned the given problem may be formulated in the following manner: the orthotropic half-plane is given ($i=0$) in the segment (a_i, b_i); on its boundary OX the loads, disturbed by regularity $q_i(x)$ ($i=1,2.....n$), move by constant velocity C is on Fig. 1.

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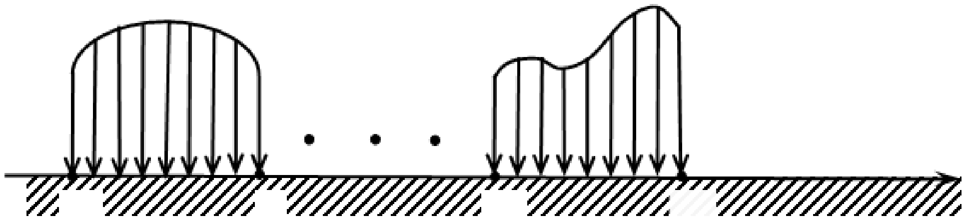

Fig.1. The loads moving by constant velocity C on the segment (a_i, b_i) of the boundary OX of orthotropic half-plane

$$\begin{cases} \dfrac{\partial \sigma_x}{\partial x} + \dfrac{\partial \tau_{xy}}{\partial y} = \rho \dfrac{\partial^2 U}{\partial t^2}, \\ \dfrac{\partial \tau_{xy}}{\partial x} + \dfrac{\partial \sigma_y}{\partial y} = \rho \dfrac{\partial^2 V}{\partial t^2}. \end{cases} \tag{1}$$

The condition of consistency:

$$\frac{1}{E_2}\left(\frac{\partial^2}{\partial x^2} - \sigma_2 \frac{\partial^2}{\partial y^2}\right)\sigma_y + \frac{1}{E_1}\left(\frac{\partial^2}{\partial y^2} - \sigma_1 \frac{\partial^2}{\partial x^2}\right)\sigma_x = \frac{2}{\mu}\frac{\partial^2 \tau_{xy}}{\partial x \partial y} \tag{2}$$

As well as zero initial conditions and the following boundary ones:

$$\left.\begin{array}{l} (\sigma_x)_{y=0} = 0 \text{ at free surface of the boundary} \\ (\tau_{xy})_{y=0} = 0, \ -\infty < x < \infty \\ (\sigma_y)_{y=0} = q_i(x), \ x \in (a_i; b_i), \ i = 1,2,\ldots,n. \end{array}\right\} \tag{3}$$

where:

E_1 and E_2 - are Young's modulus in principal directions,

σ_1 and σ_2 - Poisson's ratios,

μ - shear modulus,

U and V - the components of displacement vector which are connected which the complement of the stress tensors by the following relationship:

$$\begin{cases} \dfrac{\partial U}{\partial x} = \dfrac{1}{E_1}(\sigma_x - \sigma_1 \sigma_y), \\ \dfrac{\partial V}{\partial y} = \dfrac{1}{E_2}(\sigma_y - \sigma_2 \sigma_x), \\ \dfrac{1}{2}\left(\dfrac{\partial U}{\partial y} + \dfrac{\partial V}{\partial x}\right) = \dfrac{1}{\mu}\tau_{xy}. \end{cases} \tag{4}$$

As it well-known there are the following concepts for the components of the stress tensor:

$$\sigma_x = -2m\,\mathrm{Re}\left[(\beta_1+m)F_1(z_1)+(\beta_2+m)F_2(z_2)\right],$$
$$\sigma_y = 2n\,\mathrm{Re}\left[F_1(z_1)+F_2(z_2)\right],$$
$$\tau_y = J_m\left[aF_1(z_1)+bF_2(z_2)\right], \tag{5}$$

where:

$F_1(z_1)$ and $F_2(z_2)$ - are the analytic functions in lower half-plane,
$(y<0)$, moreover $z_1=\xi+i\beta_1\eta$, $z_2=\xi+i\beta_2\eta$,

$\xi = x-ct$, $\eta = y$, $\beta_1=\left(1-\frac{c^2}{c_1^2}\right)^{\frac{1}{2}}$, $\beta_2=\left(1-\frac{c^2}{c_2^2}\right)^{\frac{1}{2}}$

c_1 and c_2 - are the velocities of the propagation of longitudinal and cross transverse waves;
m,n,a,b - are the constants characterizing an orthotropy.

Considering the relationship (3) in (5) one can obtain:

$$J_m\left[aF_1''(\xi)+bF_2''(\xi)\right]=0\,,\ -\infty<\xi<\infty \tag{6}$$

(6) will be satisfied if we can consider that

$$F_2''(z)=-\frac{a}{b}F_1''(z) \tag{7}$$

This latter allows to express the stress components by means of one analytic function. Moreover $\mathrm{Re}\,F_1''(\xi)=0$ and $\mathrm{Re}\,F_1''(\xi)=\frac{b}{2n(b-a)}\cdot q_i(\xi)$, $\xi\in[a_i b_i]$, $i=1,2,\cdots,n$ at the free segment of the boundary.

From the last expression it is evident that the set problem is the Dirichlet's one. It may be solved by means of Schwartz's integral

$$F_1''(z)=\frac{b}{2n(b-a)}\cdot\frac{1}{2\pi i}\left[\int_{a_1}^{b_1}\frac{q_1(\xi)d\xi}{z-\xi}+\ldots+\int_{a_n}^{b_n}\frac{q_n(\xi)d\xi}{z-\xi}\right] \tag{8}$$

From the type of the loads $q_i(x)$, $x=1,..,n$ we can establish the law of stress distribution in the body by formulae (5 – 8). In Fig.2 the graphs of normal and tangent stresses are presented in the case of the various loads.

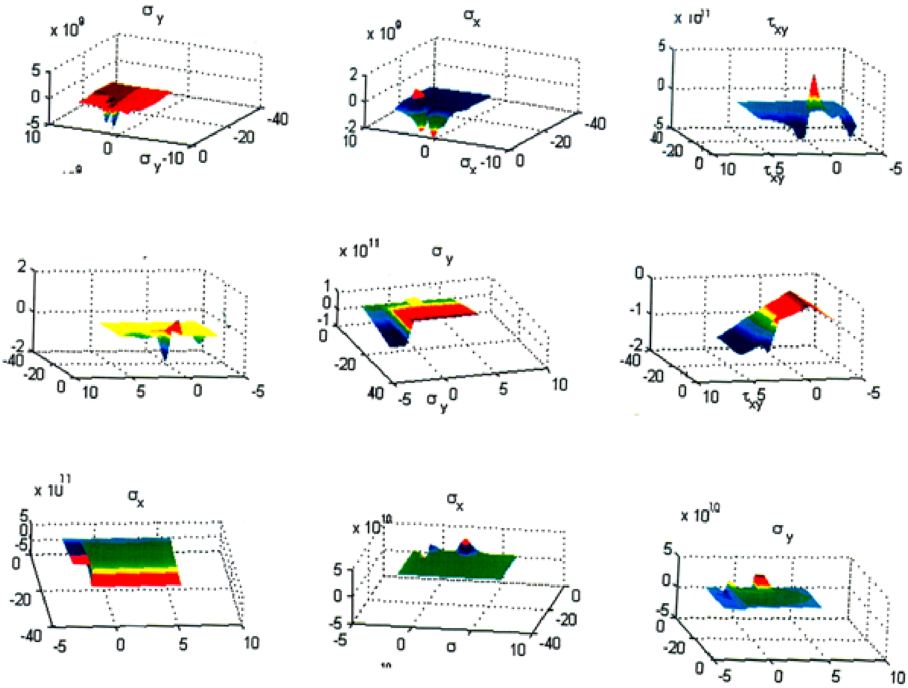

Fig.2. Dependence normal and tangent stresses on the various loads

3 CONCLUSIONS

On the basis of analysis of obtained results we may drawn the following conclusions:

- Boundary problem for the orthotropic body as well as for isotropic one is reduced to the search of two analytic functions with the boundary conditions of mixed type. The difference lies in the characteristic coefficients of anisotropy.
- Derived formulae allow to determine those sectors were the stresses attain the maximum values. Exactly these sectors are the weakest places from the viewpoint of the strength.
- Individually and in combination, normal stresses attenuate the separate sectors of the rock and in the long run together with the tangent stresses create the beneficial conditions for failure. In the course of this process the tangent stresses play a crucial role since the cracks appear at the maximum points of the tangent stresses.

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